1.1 Introduction

In the model, we considered the flow between two parallel plates separated by a distance $2H$ with a uniform heat flux imposed on both plates [1]. The fluid is driven between the plates by an applied pressure gradient in the x-direction. Assume the fluid is laminar and fully developed:

$$\frac{du}{dx} = 0, v = 0 \text{ and } w = 0$$

Then we are to determine the fully developed velocity distribution and the fully developed temperature distribution, using ANSYS FLUENT CFD Simulation.

1.1.1 Input Parameters:
Inlet velocity = 0.005m/s
Outlet Pressure = 5.00Pa
Heat Flux = 20.00W/m$^2$
Wall Thickness = 0.05m
Inlet Temperature = 300.00K
1.1.2 Basic Assumptions:
1. Steady flow
2. Constant properties
3. Newtonian fluid
4. Negligible radiation
5. Negligible gravity effects
6. Negligible viscous dissipation
7. Laminar flow
8. Fully developed
9. Negligible end effects
10. Conduction in y-direction much greater than that of x-direction

Fig. 1 – Laminar flow velocity distribution

1.2 Analytical Model

1.2.1 Conservation of mass for constant property flow

\[ \nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1) \]

For fully developed flow, \( v = w = 0 \), therefore
\[ \frac{\partial u}{\partial x} = 0 \quad (2) \]

Momentum equation for constant property flow of a Newtonian fluid:

\[ \rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mu \nabla^{2} \mathbf{V} - \nabla p + \rho g \quad (3) \]

For this case the flow is independent of the z-direction, and the flow looks exactly the same at every position along the z-axis. Hence, all derivatives in the z-direction vanish [2].

x-component for 2-D steady flow in Cartesian coordinates:

\[ \rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \left( \frac{\partial^{2} u}{\partial x^{2}} + \frac{\partial^{2} v}{\partial y^{2}} \right) - \frac{\partial p}{\partial x} + F_x \quad (4) \]

\[ \mu \left( \frac{\partial^{2} u}{\partial y^{2}} \right) = \frac{\partial p}{\partial x} \quad (5) \]

y-momentum reduces to

\[ \frac{\partial p}{\partial y} = 0 \]
Integrate twice to get:

\[ u(y) = \frac{1}{\mu} \frac{dp}{dx} \frac{y^2}{2} + A_1 y + A_2 \]  

(6)

Impose boundary conditions: \( u(-H) = 0 \) and \( u(H) = 0 \) (no slip condition at the wall)

\[ u(-H) = \frac{H^2}{2\mu} \frac{dp}{dx} - HA_1 + A_2 = 0 \]  

(7)

\[ u(H) = \frac{H^2}{2\mu} \frac{dp}{dx} + HA_1 + A_2 = 0 \]  

(8)

Adding these equations we get:

\[ A_2 = -\frac{H^2}{2\mu} \frac{dp}{dx} \quad \text{and} \quad A_1 = 0 \]

\[ u(H) = -\frac{H^2}{2\mu} \frac{dp}{dx} \left( 1 - \left( \frac{y}{H} \right)^2 \right) \]  

(9)

Thus, the velocity profile is parabolic in nature.

Recalling the definition of mean velocity \( u_m \) and mass flow rate, \( \dot{m} \):

\[ \dot{m} = \rho u_m A = \int \rho u dA \]  

(10)

\[ u_m = \frac{1}{\rho A} \int \rho u dA \]  

(11)

If the width of the flat plates perpendicular to the flow:

\[ u_m = \frac{1}{\rho(2H)W} \int_0^H \mathcal{W} \int_0^H \rho u dydz = \frac{H}{\rho} \int_0^H \frac{dp}{dx} \left( \frac{y}{H} \right)^2 - 1 \right) dy \]  

(12)

\[ u_m = \frac{H^2}{2\mu} \frac{dp}{dx} \left( \frac{y^3}{3H^2} - y \right) \bigg|_0^H = \frac{H^2}{2\mu} \frac{dp}{dx} \left( \frac{1}{3} - 1 \right) = -\frac{H^2}{3\mu} \frac{dp}{dx} \]  

(13)

These observed trends of the pressure gradient can be intuitively interpreted as follows: [3]

\[ \frac{dp}{dx} = \frac{3\mu}{H^2} u_m \]  

(14)

Hence we can establish after substitution that the velocity distribution of the fluid flow is given by:

\[ u(y) = \frac{3}{2} u_m \left( 1 - \left( \frac{y}{H} \right)^2 \right) \]  

(15)

It can be clearly seen that the velocity distribution of the flow is a parabolic curve and it is also a function of the mean velocity.

We will now decompose the Energy equation of fluid flow. We must say that the energy equation is needed for thermal analysis of the fluid [4]. The case where we do not need any thermal analysis, the energy equation is not used. This is also the case in our Simulation where we had to keep the energy equation on, to Simulate thermal properties.

1.2.2 Energy Equation for constant property flow of a Newtonian fluid:

\[ \rho c_p \left( \frac{\partial T}{\partial t} + V \cdot \nabla T \right) = k \nabla^2 T + \mu \phi + q \]  

(16)

For 2-D steady flow in Cartesian coordinates:
\[ \rho c_i \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \phi + q \]  \hspace{1cm} (17)

\[ \frac{u \frac{\partial T}{\partial x}}{\alpha} = \frac{\partial^2 T}{\partial y^2} \]  \hspace{1cm} (18)

Where we assume \( \frac{\partial^2 T}{\partial y^2} \rightarrow \frac{\partial^2 T}{\partial x^2} \)

The temperature gradient along the x-axis determines the temperature distribution along the y-axis [4]

Substitute (15) into (18)

\[ \frac{\partial^2 T}{\partial y^2} = \frac{3 u_m}{2 \alpha} \frac{dT_m}{dx} \left( 1 - \left( \frac{y}{H} \right)^2 \right) \]  \hspace{1cm} (19)

Boundary conditions for temperature can be set in any part of the calculation domain boundary to one of the following types: constant value of temperature, constant value of heat flux, heat transfer with constant value of ambient temperature and linear profile of temperature [5]:

For constant surface heat flux: \( \frac{dT}{dx} = \frac{dT_m}{dx} = \text{const} \)

Integrating (19) twice we obtain:

\[ T(y) = \frac{3 u_m}{2 \alpha} \frac{dT_m}{dx} \left( \frac{y^2}{2} - \frac{y^4}{12H^2} \right) + A_1 y + A_2 \]  \hspace{1cm} (20)

Imposing boundary conditions \( T(-H) = T(+H) = T_s \) (note that \( T_s \) is unknown):

\[ T(H) = \frac{3 u_m}{2 \alpha} \frac{dT_m}{dx} \left( \frac{H^2}{2} - \frac{H^4}{12H^2} \right) + A_1 H + A_2 = T_s \]  \hspace{1cm} (21)

\[ T(-H) = \frac{3 u_m}{2 \alpha} \frac{dT_m}{dx} \left( \frac{H^2}{2} - \frac{H^4}{12H^2} \right) - A_1 H + A_2 = T_s \]  \hspace{1cm} (22)

Which transforms to:

\[ A_1 = T_s - \frac{3H^2 u_m}{2 \alpha} \frac{dT_m}{dx} \left( \frac{5}{12} \right) \text{ and } A_2 = 0 \]

\[ T_s - T(y) = \frac{3H^2 u_m}{2 \alpha} \frac{dT_m}{dx} \left( \frac{5}{12} - \frac{1}{2} \left( \frac{y}{H} \right)^4 - \frac{1}{12} \left( \frac{y}{H} \right)^4 \right) \]  \hspace{1cm} (23)

But we know that the mean temperature is given as:

\[ T_m = \frac{1}{\rho u_m c_v} \int \rho u c_v T dA = \frac{1}{2 u_m H} \int u T dy \]  \hspace{1cm} (24)

\[ T_m = \frac{1}{u_m H} \int_{-H}^{H} \left[ \frac{3 u_m}{2} \left( \frac{y}{H} \right)^2 \right] T_s - \frac{3H^2 u_m}{2 \alpha} \frac{dT_m}{dx} \left( \frac{1}{12} \left( \frac{y}{H} \right)^4 - \frac{1}{2} \left( \frac{y}{H} \right)^2 + \frac{5}{12} \right) dy \]  \hspace{1cm} (25)

After transformation:

\[ T_s - T_m = \frac{17H^2 u_m}{35 \alpha} \frac{dT_m}{dx} \]  \hspace{1cm} (26)
Dividing (23) by (26), we have:
\[
\frac{T_s - T(y)}{T_s - T_m} = \frac{35}{136} \left[ 5 - 6\left(\frac{y}{H}\right)^2 + \left(\frac{y}{H}\right)^4 \right]
\] (27)

From (27), we can see that the temperature distribution \(T(y)\) is a function of the mean and surface temperatures. Limited comparison is made with data for flow in channels [6], this is because we are dealing with laminar flow which is a low Reynolds number flow.

Furthermore, we can theoretically calculate the Nusselt Number:

From heat transfer coefficient \(h\):
\[
h = \frac{k_f}{T_s - T_m} \left( \frac{\partial T}{\partial y} \right)_{y=H}
\] (28)

\[
N_u = \frac{hD_h}{k_f} = \frac{D_h}{T_s - T_m} \left( \frac{\partial T}{\partial y} \right)_{y=H}
\] (29)

Due to the small hydraulic diameter used in micro-channel analysis [7], the hydraulic diameter in this case is just a measure of the distance between the plates.

Where
\[
D_h = \frac{4A_s}{P} = \frac{4(2H)W}{2W} = 4H
\] (30)

\[
N_u = \frac{4H}{T_s - T_m} \frac{\partial }{\partial y} \left( T_s \left( \frac{35}{136} \left( T_s - T_m \right) \left( \frac{y}{H} \right)^4 - 6\left(\frac{y}{H}\right)^2 + 5 \right) \right)_{y=H}
\] (31)

\[
N_u = 4H \left( -\frac{35}{136} - \frac{8}{H} \right) = \frac{140}{17} = 8.2353
\] (32)

\(D_h\) is called the hydraulic diameter, where \(w\) and \(H\) had been defined in the initial diagram.

After substituting the value of \(T\) in (27) into (29), the Nusselt number is calculated to be 8.2353.

1.3 Simulation results

![Fig. 3 – Iteration graph](image1)

![Fig. 4 – Velocity vector contour](image2)
Fig. 5 – Temperature Gradient contour

Fig. 6 – Pressure Gradient contour

Fig. 7 – Pressure contour

Fig. 8 – Temperature contour

Fig. 9 – Pressure Distribution

Fig. 10 – Velocity distribution

Fig. 11 – Temperature Distribution

Fig. 12 – Velocity (u) Distribution
1.4 Analysis of Results

The following numerical results were obtained from the functional calculator of the ANSYS FLUENT CFD after Simulation.

Table 1 – Numerical Parameters/values

<table>
<thead>
<tr>
<th>Parameters/values</th>
<th>Minimum</th>
<th>maximum</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity (m/s)</td>
<td>0.000750</td>
<td>0.007500</td>
<td>0.005000</td>
</tr>
<tr>
<td>Temperature (K)</td>
<td>300.0000</td>
<td>300.0690</td>
<td>300.0180</td>
</tr>
<tr>
<td>Temperature gradient (K/m)</td>
<td>0.000000</td>
<td>26.58200</td>
<td>11.53400</td>
</tr>
<tr>
<td>Pressure gradient (Pa/m)</td>
<td>0.110000</td>
<td>8.295000</td>
<td>0.237000</td>
</tr>
</tbody>
</table>

From the table above, the minimum velocity is less than the inlet velocity, and this velocity was observed at the outlet.

The results of the table above can be directly interpreted from the various distribution contours in the Simulation results.

Thermal properties of water at 27°C are as follows:

- \( \rho = 996.5 \text{ kg/m}^3 \)
- \( k = 0.612 \text{ W/mK} \)
- \( \mu = 0.00088 \text{ kg/ms} \)
- \( Pr = 5.72 \)

Dimension of the Flat plate; Length = 60mm, Width = 14mm

To compare the analytical Nusselt number with the simulation Nusselt number, and hence to calculate the convective heat transfer coefficient and the heat flux and compare with the analytical heat flux.

The uniform surface heat flux of laminar flow in a parallel flat plate, we have the correlation formula for determining the Prandtl number as:

\[
N_u = 0.453 \sqrt{Re} \frac{1}{2} Pr^{\frac{5}{3}} \\
Re = \frac{\rho \mu D_h}{\mu} = 113.23, \quad \sqrt{Re} = 10.64
\]

Where \( D_h = 20 \text{ mm} \)

Also using the Churchill and Ozoe Correlation formula for laminar flow in parallel flat plate, we have:
The heat transfer coefficient

\[
N_{u_s} = \frac{0.886 \text{Re}_{x}^{1/2} \text{Pr}^{1/2}}{\left[1 + \left(\frac{\text{Pr}}{0.0207}\right)^{2/3}\right]^{1/4}} = 8.7
\]

The heat transfer coefficient

\[h_{x} = \frac{N_{u_s} k}{x} = 376.38 W/m^2\cdot{}K\]

Therefore the numerical Heat flux is given thus:

\[q'' = h_{x} \Delta T = 376.38(0.055) = 20.70 W/m^2\]

Where \(\Delta T\) was read directly from fig-11 (Temperature against chart count)

Since the heat flux is known, the heat transfer coefficient can be used to determine the local surface temperature.

\[T_i(x) = T_m + \frac{q''}{h_{x}} = 300 + \frac{20.5}{376.38} = 300.0559 K\]

Calculations of temperature distributions, average temperatures, and heat transfer coefficients are presented for a rectangular geometry [8]. The maximum velocity can be calculated from the velocity distribution formula, and the result compared to the one on the table above. Since we already know the mean velocity (average velocity), we also know that in this case, the velocity depends on the transverse coordinate \(y\) [9]. We can easily check the maximum velocity, which we know occurs at the central axis of the flow where \(y = 0\). Using equation (15):

\[u(y) = \frac{3}{2} u_{m} \left[1 - \left(\frac{y}{H}\right)^2\right]\]

For \(y = 0\), we have

\[u(0) = u_{\text{max}} = \frac{3}{2} u_{m} = \frac{3}{2}(0.005) = 0.0075 m/s\]

This is in good agreement with the CFD Numerical result in the table above.

The inlet pressure was also observed to be 5.05 Pa from the Simulation contour.

We can conclude that the Simulation distribution contours clearly described the temperature and velocity distribution formulae modeled in this research. This can be verified by comparing the distribution contours and the distribution formulae.

### 1.5 Conclusion

The Simulation result of Laminar flow between two parallel plates separated by a distance of 2H, often referred to as Poiseuille flow [10], agreed with the analytical functional equations of Laminar flow between two parallel plates. The Simulated results produced distribution contours that describe the temperature and velocity distribution function of the standard Mass and Energy equations of Modeled Laminar Flow. Though there was slight difference between the analytical Nusselt number and the Numerical Nusselt number, the difference can be ascribed to error due to geometrical calculation in the Reynolds number, in particular the Hydraulic diameter \(D_h\). The slight temperature change in the flow is as a result of molecular interaction between water molecules during the flow, this is the reason why the outlet temperature is almost the same with the outlet temperature. The study does not consider heat source. In the case of heat source, the temperature difference would be higher.
REFERENCES


Analysis of Fully Developed Laminar Flow between Parallel Plates with UHF Using ANSYS CFD

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Abstract – Laminar flow between two parallel placed solid plates with constant heat flux at the edges of the plates, is an idealize way of modelling the flow of coolant between parallel fuel plates commonly used in research Reactors. In this paper, we attempted to validate both the thermal and fluid properties of a fully developed laminar flow with uniform heat flux using ANSYS FLUENT. We used the transport equations to generate the velocity and temperature distribution, the pressure gradient and other profile contours. Then we used ANSYS FLUENT Simulation to generate profile contours, the results of both methods were compared and it was observed that the velocity distribution was parabolic from the Simulation, this was in agreement with the analytical result which predicted that the velocity of the fully developed Laminar flow is parabolic. Also the pressure loss and the temperature rise between the inlet and outlet flow were observed to be very small.

Keywords: Laminar, uniform, heat, flux, velocity, pressure, temperature,ANSYS, CFD, Nusselts, number